Active vibration control of beams using filtered-velocity feedback controllers with moment pair actuators

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Abstract

In this study, filtered-velocity feedback (FVF) control is proposed to stabilize a control system with a non-collocated sensor/actuator configuration. This method is applied to actively control a clamped beam with a sensor/moment pair actuator. Since the sensor/moment pair actuator is a non-collocated configuration, the control system experiences structural instability at high frequencies. Due to the roll-off property of the FVF controller, the high frequency structural instability problem can be overcome. Due to the second-order filter characteristics of the FVF controller, similar to a low pass filter, multimode disturbances can be controlled at the modes below the cut-off frequency. To verify the performance of the controller, the FVF controller is tuned to around 2 kHz, and the structural responses are successfully reduced by numerical and experimental approaches.

1. Introduction

Active vibration control (AVC) using moment pair actuators is an efficient tool for reducing vibration in the low frequency range. There are various types of active feedback vibration controls: direct velocity feedback (DVFB) and positive position feedback (PPF) are the two examples.

DVFB is a simple proportional control method based on the structural velocity feedback, so it can be implemented at a low cost. Since DVFB produces skyhook damping, it reduces response around resonance frequencies. Elliott and Balas proposed a robust AVC system using DVFB with a collocated sensor/force actuator pair.

However, there are serious instability problems in a DVFB method using moment pair actuators. The one is the non-collocated control system due to the difference between the moment pair induced locations and sensing point. Cannon reported that a non-collocated system consisting of a moment pair generated actuator (such as a PZT patch) and a feedback velocity sensor, which is located at the mid-point of the moment pair, affects the stability of the control system. The moment excitation produced by the PZT actuator couples into higher modes of the structure than lower ones, so that the sensor/actuator frequency response function has large amplitude at high frequencies where the phase exceed 90°. The non-collocation causes a phase shift due to the geometrical difference between the positions of the sensor and the actuation. Since the phase shift is proportional to the frequency, this source of instability becomes critical at a high frequency. The other problem is due to the non-duality of the velocity sensor/moment actuator pair. Because moment
excitation would require an angular velocity at the point of excitation to satisfy the dual condition. Therefore, the non-dual system has limits in the stability of the feedback loop [9]. And there are practical problems in this active control system due to wax mounted accelerometers as sensors and lightly damped structure [10]. Because wax mounted accelerometers have a mounting resonance of around 5 kHz and the phenomenon by a lightly damped internal resonance occurs at around 16 kHz. These tend to reduce the gain margin of the practical control system.

To solve this problem, Gardonio [11] proposed a phase lag compensator and applied it to a multichannel smart panel. They found that the phase lag compensator improves the stability of the control system. Hong [12] found that honeycomb structures have a low pass filter mechanism when DVFB control is implemented with the sensor and the moment pair actuator placed on the opposite side of the plate. The low pass filter mechanism leads to roll-offs at high frequencies, and thus, the control system yields the increase in the gain margin while remaining only conditionally stable. Hong [6] also proposed a solution to improve the stability using triangularly shaped PZT actuators. They placed the triangular PZT actuator at the end of a fixed beam to obtain a collocated configuration.

PPF control is an effective AVC method that helps to overcome the instability problem of the control system with the non-collocated sensor/actuator configuration. PPF is proposed to suppress the vibration of large flexible structures [13,14]. The PPF controller is an electronically implemented dynamic absorber, which is commonly used to control resonant vibration using an additional single degree of freedom system tuned to the structural resonance to be controlled. The single degree of freedom system characteristics of PPF controller gives a low pass filtering behavior, leading to rapid roll-offs in the open loop transfer function at high frequencies [15]. The PPF controller tuned to the structural resonance produces a high active damping. The PPF controller having the rapid roll-offs at high frequencies removes the control system instability due to the non-collocation of sensor/actuator. The tuning scheme of PPF controller leads to good performance, and its low pass filtering ensures the system stability.

However, PPF controllers can achieve reduction performance only at the tuning frequency. This leads to a problem that multiple PPF controllers are needed to control multimode disturbances [16]. Another problem, we should notice, is that when designing the multiple PPF controllers, a PPF controller tuned to a higher mode (e.g. the second or higher mode) influences the response at the resonances lower than the tuning frequency [17]. The influence can be recognized by active stiffness.

We, hence, consider a novel controller similar to a low pass filter, which can produce active damping at low frequencies, and, at the same time, can filter the high frequency response out. Since low pass filters cause phase distortion near the cut-off frequency, the low pass filter for the novel controller should be specially designed. The instability at high frequencies due to the non-collocation can be alleviated by using the rapid roll-off property of PPF controller, and the reduction performance in the interested frequency range can be achieved by using low pass filter property of the PPF controller, together with the concept of DVFB controller, producing the active damping.

The combination of the features of PPF and DVFB leads to a filtered-velocity feedback (FVF) control. This FVF control uses a filtered velocity signal with a low pass filter to cut off the effect of the instability sources at high frequencies. So the FVF controller produces the skyhook damping of DVFB at low frequencies, and has the roll-off property of PPF at high frequencies. In Section 2, the response of a clamped beam is formulated. In Section 3, the proposed FVF control system is mathematically formulated. In Section 4, the effect of the design parameters of the FVF controllers on the stability and performance of the controllers is examined. In Section 5, the FVF controller is implemented numerically and experimentally. Finally, conclusions are given in Section 6.

2. Response of clamped beams

Consider a clamped beam as shown in Fig. 1. The equation of motion for the system can be obtained as [18]

\[
El(1 + j\eta)\frac{\partial^4 W}{\partial x^4} + \rho A \frac{\partial^2 W}{\partial t^2} = f_p \delta(x-x_p) + \frac{\partial}{\partial x} T_s [\delta(x-x_{s2})-\delta(x-x_{s1})],
\]

where \( E \) is Young’s modulus, \( I \) is the moment of inertia, \( \rho \) is the mass density, \( A \) is the cross-sectional area and \( \eta \) is the loss factor. In this paper, \( \eta \) is expressed as an equivalent viscous damping with \( 2\zeta_{s\text{str}} \), where the modal damping ratio \( \zeta_{s\text{str}} \) is set constant for all modes. \( f_p \) is the external force applied at \( x_p \) and \( T_s \) is the control moment pair at \( x_{s1} \) and \( x_{s2} \). For harmonic

![Fig. 1. A clamped beam subjected to an external force and a moment pair.](image-url)
motions, the response, \( w \), can be expressed as
\[
W(x,t) = W(x,\omega)e^{i\omega t}.
\] (2)

Assuming that the general solution is expressed as a superposition of the modal functions
\[
W(x,\omega) = \Phi(x)p(\omega),
\] (3)
where \( p \) is the column vector of modal displacement, \( \Phi(x) \) is the row vector of the mode shape function at the location of \( x \), defined by
\[
p(\omega) = [p_1(\omega) \ p_2(\omega) \ \ldots \ p_N(\omega)]^T,
\] (4)
\[
\Phi(x) = [\phi_1(x) \ \phi_2(x) \ \ldots \ \phi_p(x)],
\] (5)

\( N \) is the number of mode superposition. Utilizing the orthogonality of the mode shape functions leads to the matrix equation as
\[
[-\omega^2 I + j \text{diag}(2\omega\omega_m + \omega_m^2) + \text{diag}(\omega_m^2)] p = \Phi(x)^T f_p + \left[ \frac{\partial \Phi(x_1)}{\partial x} - \frac{\partial \Phi(x_2)}{\partial x} \right]^T T_s,
\] (6)
where \( \omega_m \) is the \( n \)-th frequency of the structure, and \( \frac{\partial \Phi(x)}{\partial x} \) is the row vector of differentiation of the mode shape function at the location of \( x \), defined by
\[
\frac{\partial \Phi(x)}{\partial x} = \begin{bmatrix} \frac{\partial \phi_1(x)}{\partial x} & \frac{\partial \phi_2(x)}{\partial x} & \ldots & \frac{\partial \phi_n(x)}{\partial x} \end{bmatrix}.
\] (7)

The displacement response at the sensor point \((x = x_s)\) of the clamped beam subjected to an external force and moments can be obtained as
\[
W(x_s,\omega) = \Phi(x_s)p(\omega).
\] (8)
If the sensor is a velocity sensor, then the output signal, \( V \), by the structural behavior can be obtained as
\[
V(x_s,\omega) = j\omega W(x_s,\omega).
\] (9)

3. Filtered-velocity feedback control

DVFB produces skyhook damping using a structural velocity response. Therefore, it can control multimode responses. However, a DVFB controller with a moment pair actuator suffers from the instability problem caused by a phase shift due to the non-collocation between actuator and sensor. This instability generally occurs at high frequencies. We consider a new controller that retains the active damping at low frequencies and alleviates the instability at high frequencies. The concept of the novel controller is taken from the characteristics of the PPF controller. The high mode tuned PPF controller, which is designed for one specific mode, affects the system responses below the tuning mode, but does not affect the system responses above the tuning mode. This means that the high mode tuned PPF controller acts as a low pass filter. We expect that we could control multimodes by incorporating the structural velocity and the high mode tuned PPF controller.

We propose the filtered-velocity feedback (FVF) control method. The FVF controller equation can then be defined as
\[
\ddot{q} + 2\zeta\omega \dot{q} + \omega^2 q = -g\omega^2 V,
\] (10)
where \( q \) is the response of the controller, \( \zeta \) and \( \omega \) are the damping ratio and the cut-off frequency of the controller, respectively. \( g \) is the feedback gain of the FVF controller. If \( \omega \) is tuned to the structural resonance frequency, the control system works well as a modal controller at the cut-off frequency.

Fig. 2 shows the block diagram of the active control system using the FVF controller. \( V_p \) is the velocity sensor output, which represents the structural response at the sensor location due to the primary force. When the control starts, the signal is feedback to the FVF controller, \( H_{HFVF} \). Then, the controller generates the control moment, \( T_s \). In practice, \( T_s \) is proportional to the voltage signal driving the moment pair actuator. The moment pair, \( T_s \), produces the structural velocity, \( V_s \), at the sensor location. Finally, the total velocity at the sensor location, \( V_n \), which is a summation of \( V_p \) and \( V_s \), can be
obtained. Therefore, the control signal can be obtained as

$$T_s = -gH(\omega)V_r,$$

(11)

where $H(\omega)$ is the transfer function of the FVF controller, defined as

$$H(\omega) = \frac{\omega_0^2}{\omega_0^2 - \omega^2 + j2\zeta_0\omega_0\omega}.$$

(12)

Now, the plant response can be defined. It is the feedback sensor signal to the unit control action, $T_s = 1$. From Eqs. (6) and (9),

$$G(\omega) = j\omega\Phi(x_s)[-\omega^2 I + j\text{diag}(2\zeta_0\omega_0) + \text{diag}(\omega_0^2)]^{-1} \left[ \frac{\partial \Phi(x_{s1})}{\partial x} - \frac{\partial \Phi(x_{s2})}{\partial x} \right]^T.$$

(13)

Further, the transfer function of the controller, $H_{FVF}(\omega)$, can be defined. It relates the output signal, $T_s$, to the sensor signal, $V_r$:

$$H_{FVF}(\omega) = -gH(\omega).$$

(14)

The open loop transfer function can be finally obtained by considering the relationship of the signals $(V_p, V_s, \text{and } V_r)$ shown in Fig. 2

$$V_r(\omega) = V_p(\omega) + V_s(\omega) = V_p(\omega) + G(\omega)T_s(\omega).$$

(15)

Using Eqs. (11) and (14), the controlled velocity at the sensor can be written as

$$V_r(\omega) = [I + (-gH_{FVF})]^{-1}V_p.$$

(16)

For the numerical analysis in this paper, all $V_p$ will be the structural velocity due to the unit external force, $f_p$, for all frequencies at sensor location. According to the standard form of the closed loop transfer function of the negative feedback control systems, the open loop transfer function can be written from Eq. (16) as

$$\text{OLTF}(\omega) = -gH_{FVF}.$$

(17)

4. Effects of design parameters of the FVF controller

The design parameters of the FVF controller are the cut-off frequency, damping ratio and feedback gain. The design parameters significantly affect the stability and the performance of the FVF control system. A series of parametric studies in terms of the stability and performance are conducted to design the FVF controller to have a large gain margin and a good performance.

First of all, the plant response should be considered. The mechanical properties of the beam structure used in the simulation are listed in Table 1. Fig. 3 shows the plant response of the clamped beam with the velocity sensor/moment pair actuator at the sensor location. The location of the sensor/actuator is selected based on the results of work done by Hong et al. [1]. They concluded that the moment pair actuator/sensor pair should be located between 0.3\(L\) and 0.7\(L\) in order to avoid the instability at low frequencies. The phase lies between $-90^\circ$ and $90^\circ$ up to 29 kHz, while it crosses $-90^\circ$ near 29 kHz. And the response remains a small magnitude over the frequencies near 29 kHz, as shown in Fig. 3(a). These phenomena of the phase shift and the small magnitude occur due to the non-collocation between actuator and sensor. The phase critically shifts when the wavelength of the mode is shorter than the half-length of the PZT at high frequencies. The plant response encircles the $(-1,j0)$ point at high frequencies, as shown in Fig. 3(b). This implies that the system is only conditionally stable with a gain margin of $-7.93$ dB when a DVFB controller is used.

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<th>Table 1</th>
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<td><strong>Mechanical properties.</strong></td>
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Now consider an FVF controller for the parametric study. We devised the beam structure for this parametric study whose response to the primary force dominates below 2 kHz, as to be shown with solid line in Fig. 5. Then the frequency range of interest to control is considered up to 2 kHz. The initial value of the cut-off frequency can be therefore selected around 2 kHz. Since the open loop transfer function of FVF control system could be very high due to the resonance effect of the FVF controller, the cut-off frequency of the FVF controller should be a frequency where the plant response is low near 2 kHz. The cut-off frequency can be hence selected to be 1851 Hz from Fig. 3(a). With typical values of the design parameters, we can obtain the open loop transfer function rolling off at high frequencies, as shown in Fig. 4(a).

Fig. 4 also shows the variation of the open loop transfer function of the control system using FVF controller with feedback gains. The damping ratio for this calculation is 1.5. The magnitudes of the open loop transfer functions are increased as the gain is increased, while their phases remain unchanged. Therefore, the normalized Nyquist curves are completely overlapped as shown in Fig. 4(b) when they are normalized by the corresponding gain. This means that the open loop transfer function is linearly related to the gain as indicated in Eqs. (14) and (17). In terms of stability, the gain affects the stability of the system, although the phase behavior does remain constant. This is because the frequency range within the unit circle about (−1,0) in the Nyquist plane is varied with gain.

Fig. 3. The plant response of the clamped beam with the velocity sensor/moment pair actuator at the sensor location (xi = 0.7L): (a) Bode diagram and (b) Nyquist diagram.
Fig. 5 shows the variation of velocity responses at the sensor location with the gains of the FVF controller. Two key features are shown. Firstly, the responses of the closed loop control system at the feedback sensor are reduced at resonances below the cut-off frequency as the gain is increased. This can be explained by the increased active damping, as shown Fig. 4(a). Secondly, the responses at resonances above the cut-off frequency are increased as the gain is increased. This phenomenon is due to the existence of the open loop transfer function in the unit circle about the $\left(\frac{1}{C_0}, j0\right)$ point of the Nyquist diagram. However, this enhancement of the response at frequencies higher than 10 kHz is not clearly seen due to the roll-off of the FVF controller.

Fig. 6 shows the effects of the damping ratio of FVF controllers on the open loop transfer function. The gains are all set to be $8.79 \times 10^{-2}$. As shown in Fig. 6(a), the magnitude of the open loop transfer function is decreased as the damping ratio is increased around the cut-off frequency. The phase of the resonance modes near the cut-off frequency is significantly changed when the damping ratio is increased. The phase of the open loop transfer function varies in a wider frequency range with increasing damping ratio. This phase behavior can be observed more clearly in the zoomed Nyquist diagram shown in Fig. 6(b). Three Nyquist curves are shown for damping ratios of $2.0 \times 10^{-2}$, $3.0 \times 10^{-1}$ and 1.5. The Nyquist curves in the right half-plane represent the open loop transfer function below 2 kHz, and those in the left half-plane represent the open loop transfer function response above 2 kHz. However, the circles of the open loop transfer function
Fig. 5. Velocity responses of the beam structure to $f_p = 1$ (solid), and subjected to FVF controller ($f_c = 1851$ Hz, $\zeta_c = 1.5$) with the gains of $8.79 \times 10^{-2}$ (dashed), $8.79 \times 10^{-1}$ (dash-dotted) and 2.46 (dotted).

Fig. 6. Variation of the open loop transfer functions of active vibration control system using FVF controller ($f_c = 1851$ Hz, gain = $8.79 \times 10^{-2}$) with variation of damping ratios of $2.0 \times 10^{-2}$ (solid), $3.0 \times 10^{-1}$ and 1.5 (dash-dotted): (a) Bode diagram; and (b) Zoomed Nyquist diagram.
move to the third and fourth quadrants of the Nyquist diagram (rotating counterclockwise) around 2 kHz as the damping ratio is increased. This implies that the control system has the higher phase margin with the higher damping ratio. This phase shift of the open loop transfer function leads to the spill-over phenomenon.

**Fig. 7** shows the effect of the damping ratio of the FVF controller on the velocity response at the sensor location. The gain of the FVF controller is set to be $8.79 \times 10^{-2}/C_0$. It can be seen from **Fig. 7(a)** that the damping ratio of the FVF controller affects the control performance below 3 kHz while it does not above 3 kHz. Below 3 kHz, as we can see more clearly from **Fig. 7(b)**, two opposite behaviors are shown: (1) The active control system reduces the response below the cut-off frequency (1851 Hz here) but the reduction performance is decreased as the damping ratio of the FVF controller is increased. This can be explained by the clockwise rotation of the Nyquist curves as increasing the damping ratio, as shown in **Fig. 6**. The rotation of the Nyquist curve denotes the phase shift of the plant response due to the damping ratio of the FVF controller below the cut-off frequency. The increase in the phase shift as increasing the damping ratio of the FVF controller causes the active damping of the control system to be decreased. (2) On the other hand, the active system enhances the response above the cut-off frequency. But the enhancement of the performance is alleviated as the damping ratio of the FVF controller is increased. This can be explained by the existence of the Nyquist Curves in the unit circle about $(1/C_0, j0)$ and the decrease in their magnitudes when increasing the damping ratio, as shown in **Fig. 6**.

Now, we address the characteristics of the FVF control system satisfying the requirement for a proper FVF controller. The discussion is made on the open loop transfer function of the FVF control system with a high damping ratio, 1.5 in this case, as shown with the dashed line in **Fig. 4**. Due to the high damping of the FVF controller, its resonance effect on the magnitude adjacent to the cut-off frequency is almost disappeared, but on the phase is significantly appeared. Then the magnitude of the open loop transfer function around the cut-off frequency do not show the resonance effect as shown in **Fig. 4**. It is noted, on the other hand, that its phase is shifted in a wide frequency range, as shown in **Fig. 6(a)**. In the Nyquist diagram shown in **Fig. 6(b)**, the response circle of the modes just below the cut-off frequency rotates counterclockwise due to the high damping ratio of the FVF controller. At frequencies higher than the cut-off frequency, the magnitude of the open loop transfer function decreases as the frequency is increased. This roll-off property in high frequencies is due to the characteristics of the FVF controller formulated by the second-order ordinary differential equation which leads to the second-order low pass filter. Since the high frequency roll-off property can alleviate the potential instability in high frequencies due to the non-collocated sensor/actuator, a high gain margin can be obtained. The phase shifted from the
cut-off frequency, which causes the Nyquist curve in the higher frequency ranges above the tuning mode to lie in the left half-plane. This means that the modes adjacent to the cut-off frequency to become conditionally stable. Note that the amplitude of the modes adjacent to the cut-off frequency does not reduce much, but the phase does change a lot, as shown in Fig. 4(a). Therefore, the closed loop velocity in these higher modes will be enhanced. The open loop transfer function response at the modes just below the cut-off frequency is decreased and rotates counter-clockwise due to the high damping ratio of the FVF controller, as shown in Fig. 4(b). As a result, this control system can obtain the gain margin of 9.69 dB.

In summary, to design an FVF controller for controlling the multimodes at frequencies below the cut-off frequency, a high damping ratio is required to alleviate the effects of the mode at the cut-off frequency on the instability of the control system. A high gain is also required to compensate for the effects of the damping on the low frequency response. The high gain of the FVF controller leads to reduction of the response at the modes below the cut-off frequency.

5. Implementation of the FVF controller

Based on the results of the parametric study, the FVF controller is implemented numerically and experimentally. The numerical FVF control system is implemented as the block diagram shown in Fig. 2. Also the experimental FVF control system is implemented, as shown in Fig. 8, in order to verify the stability and the performance of the FVF controller for active control of an aluminum beam using PZT actuator.

The aluminum beam with a length of 0.5 m is fixed at both ends. A force transducer, B&K 8200, and a mini-shaker, B&K 4810, are located at 0.2L. An accelerometer, B&K 4393, and a piezoceramic actuator are attached to the beam at 0.7L. The piezoceramic actuator is a rectangular shaped 1–3 mode PZT patch (25 mm × 25 mm × 1 mm) whose material properties hold with PZT4 series, manufactured by Kyungwon Ferrite Ind. Co., Ltd. (South Korea). The primary disturbance field of vibration is generated by a mini-shaker with a power amplifier, B&K 2706. A signal generator, a built-in module of B&K Pulse, is connected to the power amplifier. A random signal is used in this experiment for the primary disturbance. A force transducer is installed between the mini-shaker and the beam to measure the primary force. The measured primary force is used for normalizing the primary field of vibration. To do this, the force transducer is connected to a conditioning amplifier, B&K 2692, and then to the input channel of the signal analyzer, B&K Pulse. The disturbances are measured by the

![Diagram](image-url)
accelerometer. The signal from the acceleration is conditioned and integrated to the velocity signal by using a conditioning amplifier, B&K 2635, and this velocity signal is feedback to the DSP, dSPACE DS1103. The DSP generates a control signal that is input to the power amplifier, PCB AVC 790, to drive the PZT patch control actuator. The DSP is programmed to work as the FVF controller. This practical FVF controller is implemented by programming the transfer function

$$H_{FVF}(s) = \frac{-\omega_c^2}{s^2 + 2\zeta_c \omega_c s + \omega_c^2},$$

(18)

which is a rewritten form of Eq. (12) in the s-domain. Eq. (18) is coded using Matlab/Simulink. The transfer function, Eq. (18), is downloaded to the ROM of the DSP for practical implementation. The DSP then works as the FVF controller. The sampling frequency of the DSP is set to be 200 kHz.

The numerical implementation of the FVF control is achieved with the values of the design parameters: the cut-off frequency of 1851 Hz, the feedback gain of $8.79 \times 10^{-1}$, and the damping ratio of 1.5. Note that they are not optimal in terms of stability and performance. They are selected based on the parametric study however. We first select the cut-off frequency of the FVF controller from the plant response shown in Fig. 3 to be a frequency near the maximum frequency of interest (2 kHz in this case) at which the plant response is low. In addition, the feedback gain and the damping ratio are determined taking into account of the influence of the phase shift and the high frequency roll-off.

The experimental implementation of the FVF is also achieved by the same procedure as that in the numerical implementation. We measured firstly the plant response of the clamped beam with the sensor/moment pair actuator at $x_s = 0.7L$. Fig. 9 shows the measured plant response. The magnitude of the plant response increases as the frequency increases, as shown in Fig. 9(a). This trend is due to the moment pair actuation, which couples well into higher modes. The Nyquist diagram in Fig. 9(b) shows that the plant response exists in the right half-plane below 15.8 kHz, while the response above 15.8 kHz move to the left half-plane slowly (rotating clockwise). Therefore, this system is conditionally stable. Now consider the cut-off frequency for controlling the vibration below 2 kHz as we did in the numerical implementation. We examine the plant response, around 2 kHz, find a low magnitude in the plant response and set the corresponding frequency to be the cut-off frequency (1920 Hz in this case). We then implement the FVF controller with the same design parameters which are the same damping ratio ($\zeta_c = 1.5$) as in the numerical implementation. The feedback gain is initially set to be 1.0.

![Fig. 9. The measured plant response of the clamped beam with the sensor/moment pair actuator pair at 0.7L: (a) Bode diagram and (b) Nyquist diagram.](image-url)
Fig. 10 shows the transfer functions of the FVF controllers. The numerical FVF controller reveals three features: no resonant peak, a high frequency roll-off, and the phase staying between 180° and 0°. Note that the removal of the resonant peak is achieved by a high damping ratio, but it causes the high frequency roll-off to be reduced and it makes the phase shift to be occurred in a wider frequency range. The magnitudes of the transfer functions of the numerical and experimental FVF controllers with the damping ratio of 1.5 are well matched, but the phases are not. The phase of the experimental FVF controller shifts as the frequency increases more rapidly than that of numerical FVF controller. This is because the sampling frequency is not high enough to convert digital signals into analog signals. For this conversion, the

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**Fig. 10.** Transfer functions of the FVF controllers with a gain of 1, for a comparison of the numerically implemented FVF controller tuned to 1851 Hz with a damping ratio of 1.5 (solid) with the experimentally implemented FVF controller tuned to 1920 Hz with damping ratios of 1.5 (dashed) and 12 (dash-dotted).

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**Fig. 11.** The measured open loop transfer function of the FVF control system tuned to 1920 Hz with a gain of 5000 and damping ratio of 12: (a) Bode diagram and (b) Nyquist diagram.
sampling frequency must be at least 10 times the maximum frequency of interest. This property of the requirement for
D/A conversion gives rise to additional phase shifts. In the FVF control system, the phase property of the controller is very
important, because it leads active control signals. We therefore carefully adjust the damping for the experimental FVF
controller to have its phase to be from 0° to 180° below 30 kHz, as denoted by dash-dotted line in Fig. 10. This is the similar
feature to the phase of the numerical FVF controller. So, we selected the damping ratio of 12 to recover the additional
phase shift in the experimental implementation.

The stability of the experimental FVF controller is examined by the measured open loop transfer function shown in
Fig. 11. We can obtain qualitatively the same features as those from numerical stability analysis of the FVF control system
described in Section 4. The design parameters of the experimental FVF controller are cut-off frequency of 1920 Hz,
damping ratio of 12, and feedback gain of 5000. It can be seen from Fig. 11(a) that the magnitudes of the open loop transfer
function are quite high up to the cut-off frequency. Also they tend to decrease as the frequency increases, above the cut-off
frequency. This leads to big Nyquist circles at low frequencies and small circles at high frequencies. Under the feedback
gain of 5000, as shown in Fig. 11(b), the Nyquist curves do not encircle \((-1,0)\). Hence, the experimental FVF controller is
conditionally stable and can produce a high active damping at low frequencies.

We finally evaluated the performance of the FVF control system experimentally. Fig. 12 shows the measured velocity
response of the clamped beam at the sensor location. Using the experimental FVF controller, the responses below the cut-
off frequency are successfully reduced by more than 10 dB. Consequently, the FVF controller proposed in this study can be
applied for multimode control while alleviating the high frequency instability.

6. Conclusion

This study investigates the active vibration control of clamped beams using FVF controllers with a non-collocated sensor/
moment pair actuator configuration. A moment pair actuator is used for the control actuator and a velocity sensor is used for the
feedback sensor, located at the opposite center of the moment pair actuator. This non-collocated configuration is the main
source of instability problems in control systems. In order to overcome such instability problems, the FVF controller is proposed.
To obtain the characteristics of the FVF controller, a parametric study followed by stability and performance analysis is
conducted for a proposed design of the FVF controller. The following conclusions are highlighted:

The parametric study on the design parameters of the FVF controller is conducted to characterize their effects on the
stability and performance of the controller. It is found that, as any conditionally stable system, the gain does not affect
the absolute stability, but does affect the relative stability. As the damping ratio of the FVF controller is increased, the
magnitude of the open loop transfer function decreases around the cut-off frequency, and the phase of the open loop
transfer function is affected over a wider range near the cut-off frequency. This means that the control system can be
stabilized with the increase in the damping ratio of the FVF controller.

FVF controllers are numerically and experimentally implemented based on the results of the parametric study. Comparisons
are made to verify the performance of the controller, and the qualitative features agree well. Using an FVF controller, the source
of instability at high frequencies is reduced, and therefore, the FVF controller can reduce the responses below the cut-off
frequency, including more than 10 dB reductions of the second and third modes. Consequently, the FVF controller proposed in
this study can be applied for multimode control while can alleviate the high frequency instability.

Acknowledgement

This work was supported by the 2009 Research Fund of Ulsan College.
References